GIS AND MULTICRITERIA DECISION ANALYSIS

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Chapter

3

Introduction to Multicriteria Decision Analysis

In this chapter we focus on multicriteria decision making (MCDM). The terms multicriteria decision analysis *and* multicriteria decision analysis *(MCDA) are used interchangeably. Broadly speaking, MCDM problems involve a set of alternatives that are evaluated on the basis of conflicting and incommensurate criteria.* Criterion *is considered a generic term that includes both the concepts of attribute and objective. Accordingly, two broad classes of MCDM can be distinguished: MADM (multiattribute decision making) and MODM (multiobjective decision making). Both MADM and MODM problems are further categorized into single-decision-maker problems and group decision problems. These two categories are, in turn, subdivided into deterministic, probabilistic, and fuzzy decisions. Deterministic decision problems assume that the required data and information are known with certainty and that there is a known deterministic relationship between every decision and the corresponding decision consequence. Probabilistic analysis deals with a decision situation under uncertainty about the state of problem's environment and about the relationships between the decision and its consequences. Whereas probabilistic analysis treats uncertainty as randomness, it is also appropriate to consider inherent imprecision of information involved in decision making; fuzzy decision analysis deals with this type of uncertainty. Conventional MCDM techniques have largely been aspatial in the sense that they assume a spatial homogeneity within the study area. This assumption is unrealistic in many decision situations because the evaluation criteria vary across space. Consequently, there is a need for an explicit representation of the geographical dimension in MCDM. The second part of this chapter provides a framework for GIS-based (or spatial) multicriteria decision analysis. The framework integrates the GIS capabilities of data acquisition, storage, retrieval, manipulation, and analysis and the capabilities of MCDM techniques for aggregating the geographical data and the decision maker's preferences into unidimensional values of alternative decisions.*

3.1 ELEMENTS OF MULTICRITERIA DECISION ANALYSIS

A number of approaches to structuring MCDM problems have been suggested in the decision analysis literature (Keeney and Raiffa 1976; Saaty 1980; Chankong and Haimes 1983; Kleindorfer et al. 1993). In general, MCDM problems involve six components: (1) a *goal* or a set of goals the decision maker (interest group) attempts to achieve; (2) the *decision maker* or group of decision makers involved in the decision-making process along with their preferences with respect to *evaluation criteria;* (3) a set of evaluation criteria (*objectives* and/or *attributes*) on the basis of which the decision makers evaluate alternative courses of action; (4) the set of decision *alternatives,* that is, the decision or action variables; (5) the set of uncontrollable variables or *states of nature* (decision environment); and (6) the set of *outcomes* or consequences associated with each alternative–attribute pair (Keeney and Raiffa 1976; Pitz and McKillip 1984). The relationships between the elements of MCDM are shown in Figure 3.1. The central element of this structure is a *decision matrix* consisting of a set of columns and rows (Pitz and McKillip 1984). The matrix represents the decision outcomes for a set of alternatives and a set of evaluation criteria.

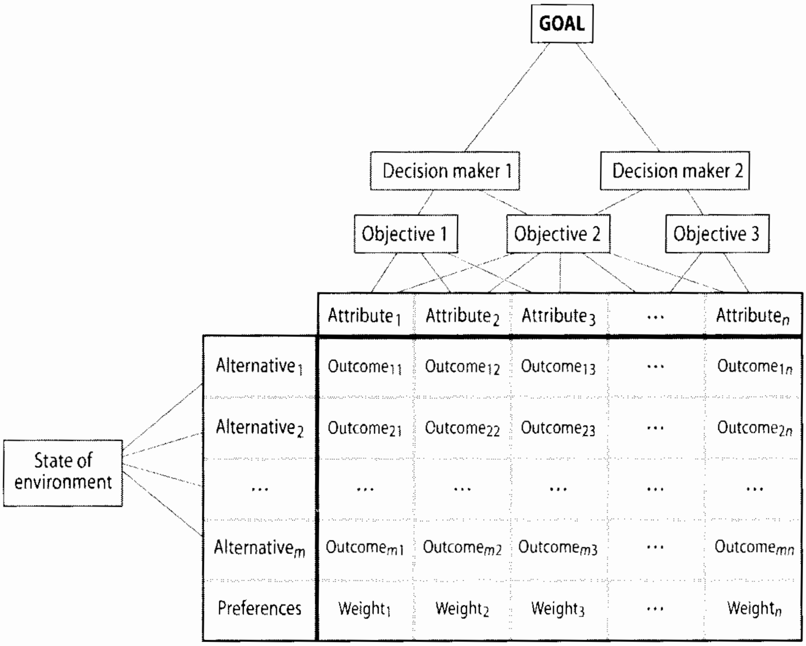

FIGURE 3.1 Framework for multicriteria decision analysis.

The structure of the columns consists of levels representing the decision makers, their preferences, and evaluation criteria. These elements are organized in a hierarchical structure. The most general level is a goal. At this level a desired end state resulting from decision-making activity is specified. For example, in the context of land-use planning, the goal may be to improve quality of life in a particular region. Complex decision problems typically involve a number of decision makers (interest groups). A decision maker may consist of a single person or a group of people, such as government or corporate organizations. The decisions require analysis of the values of persons affected by the decision, who are often characterized by unique preferences with respect to the relative importance of criteria on the basis of which alternative decisions are evaluated. The preferences are typically operationalized in terms of weights assigned to the evaluation criteria. A *criterion* is a standard of judgment or a rule to test the desirability of alternative decisions (Hwang and Yoon 1981). It is a generic term that includes both objectives and attributes. Any multiple criteria decision problem involves a set of objectives, a set of attributes, or both. Although in real-world decision problems the objectives and attributes are often involved in a mixed fashion, the distinction between these two concepts is of crucial importance for an understanding of the nature and essence of MCDM approaches. An objective is a statement about the desired state of a spatial system (e.g., a desired pattern of land use). The objectives are made operational by assigning to them one or more attributes (see Section 3.2 for a detailed discussion).

The rows of the decision matrix represent decision alternatives (Figure 3.1). All decisions are made in some kind of environmental context and therefore involve many factors beyond the control of the decision maker. These uncontrollable factors are referred to as *states of nature* or *states of environment.* Note that the term *nature* as used here refers to the general unpredictability of the decision-making environment. A state of nature can be a state of the economy (e.g., recession, inflation), a weather condition (rain, drought, frost), an action of a competitor, or other situations over which the decision maker has little or no control, and therefore they must be included in the unpredictability of nature. Each state is assumed to be independent of other states and immune to manipulation by the decision maker; that is, the decision environment is neutral. Also, it is assumed that a finite number of possible states of nature can occur. The states of nature reflect the degree of uncertainty about decision outcomes (consequences). Therefore, for each decision alternative there is a set of possible outcomes. Which outcome will actually follow a decision depends on the state of nature. If only one state of nature is considered, only one decision outcome is associated with a given alternative. This situation is represented in Figure 3.1.

The decision outcomes depend on the set of attributes for evaluating alternatives. Consequently, an entry in the intersection of each row and column of the decision matrix is the decision outcome associated with a particular alternative and attribute. The matrix cells contain a single entry if a single state of nature is considered, and they contain a number of outcomes if the decision situation requires consideration of more than one state of nature. Thus the decision outcomes in each row of the matrix are represented as the attribute levels, which measure the degree of achievement or performance of a decision alternative. The decision problem requires that the set of outcomes are ordered so that the best alternative can be identified.

3.2 CLASSIFICATION OF MULTICRITERIA DECISION PROBLEMS

MCDM problems can be classified on the basis of the major components of multicriteria decision analysis presented in Section 3.1. Three dichotomies can be distinguished: (1) multiobjective decision making (MODM) versus multiattribute decision making (MADM), (2) individual versus group decision-maker problems, and (3) decisions under certainty versus decisions under uncertainty. This classification is shown in Figure 3.2. The distinction between MADM and MODM is based on the classification of evaluation criteria into attributes and objectives.

These two approaches can be further subdivided into two categories depending on the goal-preference structure of the decision maker. If there is a single goal-preference structure, the problem is referred to as individual decision making, regardless of the number of decision makers actually involved. On the other hand, if individuals (interest groups) are characterized by different goal-preference structures, the problem becomes that of group decision making. The subdivision of decision problems into individual and group decision making applies to both MADM and MODM.

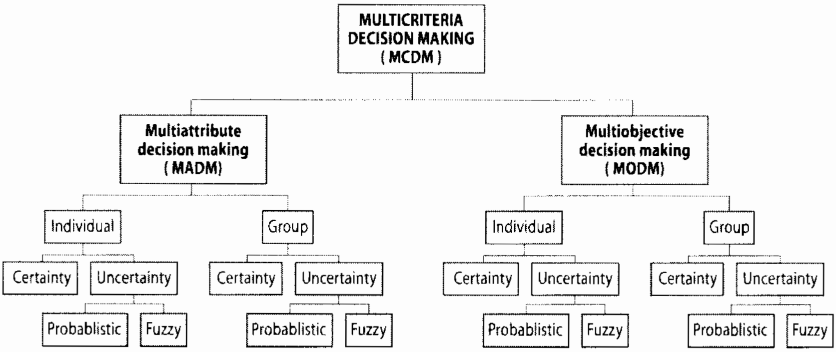

FIGURE 3.2 Classification of multicriteria decision problems.

Finally, decision problems can be categorized into decisions under certainty and decisions under uncertainty, depending on the amount of information (knowledge) about the decision situation that is available to the decision maker and analyst. If the decision maker has perfect knowledge of the decision environment, the decision is made under conditions of certainty. Most real-world decisions involve some aspects that are unknowable or very difficult to predict. This type of decision making is referred to as decisions under conditions of uncertainty. We have to recognize, however, that uncertainty may come from various sources. To this end, the decision under uncertainty may be further subdivided into two categories: probabilistic and fuzzy decision making.

3.2.1 Multiobjective Versus Multiattribute Analysis

As suggested earlier, criteria are the standards of judgment or rules on the basis of which the alternative decisions are ranked according to their desirability. *Criterion* is a generic term including the concepts of attribute and objective. Thus *MCDM* is used as the blanket term, which includes both multiobjective and multiattribute decision making.

Attributes are the properties of elements of a real-world geographical system. More specifically, an attribute is a measurable quantity or quality of a geographical entity or a relationship between geographical entities. In the context of a decision-making problem, the entities and the relationships are referred to as the *objects of decisions*. We assume that decisions are made to change or leave unchanged the state of a spatial system, that is, the state of entities and the relationships among them. The concept of attribute is synonymous with the often-used concept of the measurement of system (or system element) performance. An attribute is used to measure performance in relation to an objective. It can be thought of as the means or information sources available to the decision maker for formulating and achieving the decision maker's objectives (Starr and Zeleny 1977).

An *objective* is a statement about the desired state of the system under consideration. It indicates the directions of improvement of one or more attributes. Objectives are functionally related to, or derived from, a set of attributes. For any given objective, several different attributes might be necessary to provide complete assessment of the degree to which the objective might be achieved. For example, if we have the objective "minimizing the population exposure to air pollution," we may use the attribute "number of people exposed to sulfur oxides above a specified standard" (e.g., 80 $\mu g/m^3$ not to be exceeded more than once per year), and "number of people exposed to carbon monoxide above a specified standard" (e.g., 100 mg/m^3 not to be exceeded more than once per year).

Table 3.1 provides a comparison of MODM and MADM approaches. The MODM problems require that means–ends relationships be specified, since

TABLE 3.1 Comparison of MODM and MADM Approaches

	MODM	***MADM***
Criteria defined by:	Objectives	Attributes
Objectives defined:	Explicitly	Implicitly
Attributes defined:	Implicitly	Explicitly
Constraints defined:	Explicitly	Implicitly
Alternatives defined:	Implicitly	Explicitly
Number of alternatives	Infinite (large)	Finite (small)
Decision maker's control	Significant	Limited
Decision modeling paradigm	Process-oriented	Outcome-oriented
Relevant to:	Design/search	Evaluation/choice
Relevance of geographical data structure	Vector-based GIS	Raster-based GIS

Sources: Hwang and Yoon (1981, Table 1.1, p. 4) and Starr and Zeleny (1977).

they deal explicitly with the relationship of attributes of alternatives to higher-level objectives of the decision maker. Therefore, this category of multicriteria approaches involves designing the alternatives and searching for the "best" decisions among an infinite or very large set of feasible alternatives. The role of MODM approaches in decision making is to provide a framework for designing a set of alternatives. Each alternative is defined implicitly in terms of the decision variables and evaluated by means of objective functions. If there is a direct correspondence between attributes and objectives, the multiobjective problem becomes a multiattribute decision problem. Multiattribute decision problems require that choices be made among alternatives described by their attributes. This implies that attribute–objective relationships are specified in such a form that attributes are regarded as both objectives and decision variables. The set of attributes is given explicitly. Attributes are used as both decision variables and decision criteria.

MODM recognizes that attributes of alternatives are often just means to higher ends, the decision maker's objectives. While MADM methods obtain preferences, usually in the form of function forms and weights, directly for levels on the attributes, MODM methods derive these from the preferences among objectives and the functions relating attributes to objectives (MacCrimmon 1973). An attribute is a concrete descriptive variable; an objective is a more abstract variable with a specification of the relative desirability of the levels of that variable. MADM problems are assumed to have a predetermined, limited number of alternatives. Solving an MADM problem is a selection process, as opposed to a design process. The MODM problem is continuous in the sense that the best solution may be found anywhere within the region of feasible solutions. Therefore, MADM and MODM problems are sometimes referred to as *discrete* and *continuous* decision problems, respectively (Hwang and Yoon 1981).

3.2.2 Individual Versus Group Decision Making

Many spatial decisions are made by groups (multiple decision makers rather than an individual decision maker (Hwang and Lin 1987; Massam 1988). Group decision-making problems are encountered frequently in the public sector. For example, major decisions of locating public "goods" (e.g., the location of a hospital, school, park) and public "bads" (e.g., the location of noxious facilities such as a hazardous waste incinerator or waste landfill) require an analysis of the values of different interest groups, that is, people affected by the "goods" and "bads." Similarly, land-use decisions are typically complex, owing to the unavoidable trade-offs inherent in protecting or developing specific lands and the differential impacts on various stakeholder groups. An environmental conflict arises whenever the activities of one sector reduce the capability of the land for other stakeholder activities. Stakeholders' values and interests have to be analyzed to determine the land-use pattern that maximizes consensus or minimizes conflict. The degree of consensus can be considered as a major determinant of the nature of the choice (decision) process, and therefore of how choice should be organized (Massam 1993). Consequently, the distinction between individual and group decision making rests less on the number of people involved than on the consistency of the group's goals, preferences, and beliefs. If we can assume a single goal–preference–belief structure, we are dealing with individual decision making, regardless of the number of people actually involved. On the other hand, if any of these components varies among those constituting the decision-making group, we are coping with group choice making.

In the context of multiple decision makers, it is useful to make a distinction between a team and a coalition (Rothenberg 1975). A *team* is defined as a group of people if it is characterized by *a mutually consistent set of preferences;* that is, all persons have the same preference orderings on all outcomes that are relevant to the decisions. In this case, even though many people are involved in making a decision, a single decision model and analysis is possible. Everyone on the team must agree on a unitary perspective for a particular decision. By contrast with the team, a *coalition* is made up of people who compromise "their partly similar, partly divergent outlooks" (Rothenberg 1975, p. 63). Coalition participants can agree on the structure of the problem (the set of alternatives and evaluation criteria) but disagree on the *relative importance of the evaluation criteria.* This means that the problem can be structured in terms of a single model. It requires, however, multiple analysis because the various preferences lead to various orderings of the alternatives. Thus teams and coalitions represent two decision situations in which a single problem model can be used. The two groups differ in that decisions made by coalitions require multiple analysis to accommodate the various preferences of the coalition participants, whereas a single analysis is possible for team members.

If the individual decision makers disagree on a set of alternatives and/or evaluation criteria to be considered for the decision problem, they must be regarded as participating in multiple and separate decisions. To this end, two forms of multiple decisions can be distinguished: *competitive* decision making, which includes situations in which some sort of conflict exists among decision makers, and *independent* decision making, which involves situations in which the various decision makers are independent of each other, although the action of any one person or group may have important consequences for the others. Competitive decision making requires multiple analyses, but it can be structured in terms of a single model. The difficulty involved in competitive decision making is the selection of the appropriate perspective for the analysis (Pitz and McKillip 1984).

3.2.3 Decision Making Under Certainty Versus Uncertainty

Broadly speaking, there are two sources of uncertainty involved in making a decision. The first concerns the validity of information (Keeney and Raiffa 1976). The decision maker may be unsure whether the information about the spatial problem is error-free and appropriate for predicting the outcome of any decision made. The second source of uncertainty concerns future events that might lead to differentially preferred outcomes for a particular decision alternative. In a sense, the former is a special case of the latter. For example, in the context of a farmer's spatial decision, the uncertainty may be related to the question of where and what kinds of agricultural production should be practiced on the farm. Since the yield is influenced by weather conditions, which may be unpredictable, the farmer faces a decision under uncertainty. The limited (uncertain) information about future weather conditions makes any prediction prone to error. Similarly, spatial decisions concerning location or relocation of a retail facility are surrounded by uncertainty because of the unpredictability of the locational decisions of competitors. Each competitor has its own locational strategy, which may be difficult to predict because of imperfect information about the decision situation.

Although uncertainty exists in many decision situations, the amount of uncertainty (or the amount of information about the decision problem) varies greatly. To this end, it is useful to locate a decision problem on a continuum ranging from a predictable situation to one that is extremely difficult to predict. The former is referred to as a *deterministic situation* (or *certainty*); the latter is referred to as a *decision problem under uncertainty*. Accordingly, MCDM problems can be classified into two categories: MCDM under certainty and MCDM under uncertainty. In a decision under certainty it is assumed that all relevant information about the decision situation is known and that there is a known deterministic connection between every decision and the corresponding outcome. This means that under conditions of certainty only one state of nature is possible or, alternatively, any variation that is possible will not affect the consequences of choosing a particular option. Either way, the decision is judged to be insensitive to any uncontrollable factors present.

Some decision situations come close to the case of certainty; that is, the uncertainty is so remote that it can be disregarded as a factor. Indeed, many spatial problem formulations assume that the future state of nature is known with certainty. Such secondary attention to uncertainty (risk) factors is often a necessity because of data availability or costs. Thus, even when uncertainty is recognized, it may have to be ignored because of insufficient data for evaluation or because the evaluation would require too much time or money. Moreover, a decision maker (or analyst) can deliberately choose to model a decision as occurring under a condition of certainty if it is believed that modeling it in a probabilistic manner will add nothing to the analysis of the problem. It may be a perfectly legitimate ploy to assume, for example, that population figures by region will have a certain value and to assume that the investment costs of establishing a facility in alternative locations will take a certain level, even though we know that these figures are merely best guesses. This does not imply, however, that deterministic decision problems may be particularly easy or straightforward. The problems may be complex because a multitude of alternative strategies may be present, which may be evaluated on the basis of incommensurate and conflicting criteria by a number of interest groups or decision makers. Furthermore, to deal with the uncertainty involved in a deterministic problem formulation, sensitivity analysis can be performed to demonstrate the possible outcomes under different scenarios. It is argued that sensitivity analysis is a critical element of any spatial decision problem (see Chapter 8 for a detailed discussion).

Two basic types of uncertainty may be present in a decision situation: (1) uncertainty associated with limited information about the decision situation, and (2) uncertainty associated with fuzziness (imprecision) concerning the description of the semantic meaning of the events, phenomena, or statements themselves. Consequently, both MADM and MODM problems under uncertainty can be subdivided further into *probabilistic* (stochastic) and *fuzzy* decision-making problems, depending on the type of uncertainty involved. The probabilistic decisions have a stochastic character. They are handled by probability theory and statistics. The concept of uncertainty can be treated as secondary to that of probability. Once the probability of the event concerned is known, the quantitative aspect of the uncertainty is determined. The precise nature of the link will depend on the view actually taken.

In many cases the uncertainty is not due to randomness but to some imprecision whose formal treatment cannot be handled by probability theory. Note that the outcome of a stochastic event is either true or false. However, in a situation where the event itself is ambiguous, the outcome may be given by a quantity other than true (1) or false (0). The problem of ambiguity can be structured as the degree to which an event "more or less belongs" to a class.

This type of situation is handled by the fuzzy set theory. Specifically, the theory of fuzzy sets provides a natural basis for the *theory of possibility*, playing a role similar to that of measure theory in relation to the theory of probability (Zadeh 1965). It is important to realize that possibility theory is an alternative information theory to that based on probability. Although possibility theory is logically independent of probability theory, they are related: both arise in Dempster–Shafer evidence theory as fuzzy measures defined on random sets (Klir and Yuan 1995). Furthermore, possibility theory directly generalizes both nondeterministic process theory and interval analysis [see Eastman (1997) for a discussion on spatial aspects of the possibility theory].

3.3 FRAMEWORK FOR SPATIAL MULTICRITERIA DECISION ANALYSIS

At the most rudimentary level, a spatial multicriteria decision problem involves a set of geographically defined alternatives (events) from which a choice of one or more alternatives is made (their ordering performed) with respect to a given set of evaluation criteria (Carver 1991; Heywood et al. 1995; Jankowski 1995; Keller 1996; Malczewski 1996). The alternatives are defined geographically in the sense that results of the analysis (decisions) depend on their spatial arrangement. In GIS terminology, the alternatives are a collection of point, line, and areal objects, attached to which are criterion values (see Chapter 2). Conventional MCDM techniques have largely been aspatial. They typically use average or total impacts that are deemed appropriate for the entire area under consideration (Tkach and Simonovic 1997). In other words, conventional approaches assume a spatial homogeneity within the study area. This assumption is clearly unrealistic in many decision situations because the evaluation criteria vary across space. Spatial multicriteria analysis represents a significant departure from the conventional MCDM techniques because of its explicit geographic component. In contrast to the conventional MCDM, spatial multicriteria analysis requires both data on criterion values and the geographical locations of alternatives. The data are processed using GIS and MCDM techniques to obtain information for making the decision. Consequently, the terms *GIS-based multicriteria decision analysis* and *spatial multicriteria analysis* will be used interchangeably.

Spatial multicriteria decision analysis can be thought of as a process that combines and transforms geographical data (input) into a resultant decision (output) (Figure 3.3). The MCDM procedures (or decision rules) define a relationship between the input maps and the output map. The procedures involve the utilization of geographical data, the decision maker's preferences, and the manipulation of the data and preferences according to specified decision rules. They aggregate multidimensional geographical data and information into unidimensional values of alternative decisions. The critical aspect of spatial

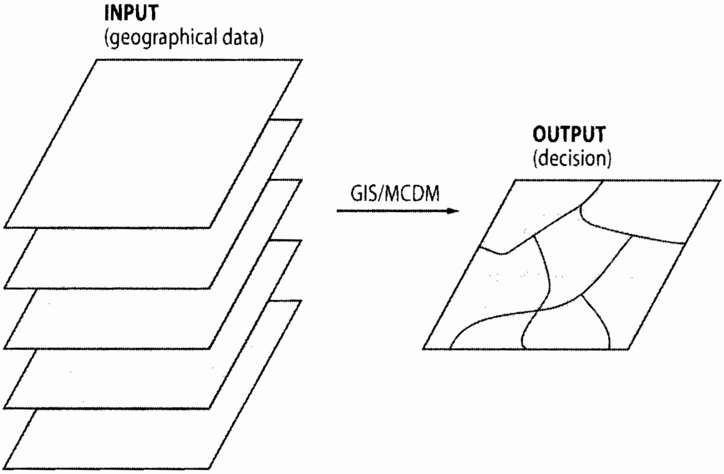

FIGURE 3.3 Spatial multicriteria decision analysis: input–output perspective.

multicriteria analysis is that it involves evaluation of geographical events based on the criterion values and the decision maker's preferences with respect to a set of evaluation criteria. This implies that the results of the analysis depend not only on the geographical distribution of events (attributes) but also on the value judgments involved in the decision-making process. Accordingly, two considerations are of critical importance for spatial multicriteria decision analysis: (1) the GIS capabilities of data acquisition, storage, retrieval, manipulation, and analysis, and (2) the MCDM capabilities for aggregating the geographical data and the decision maker's preferences into unidimensional values of alternative decisions. The large number of factors necessary to identify and consider in making spatial decisions and the extent of the interrelationships among these factors cause difficulties in decision making. The difficulty is that in attempting to acquire data and to process the data to obtain information for making decisions, the complexity of the problem may require processing at a level that exceeds a decision maker's cognitive abilities. To this end, the role of GIS and MCDM techniques is to support the decision maker in achieving greater effectiveness and efficiency of decision making while solving spatial decision problems. It is argued that the combination of GIS capabilities with MCDM techniques provides the decision maker with support in all stages of decision making, that is, in the intelligence, design, and choice phases of the decision-making process (see Section 2.3).

3.3.1 Formal Structures

Based on the general classification of MCDM problems (see Section 3.2), the spatial multicriteria decision problems can be subdivided into two fundamental categories: *spatial multiattribute* and *spatial multiobjective decisions* (Malczewski 1999). The two categories are also referred to as spatial MADM and spatial MODM, respectively.

The MADM approaches assume that the set of alternatives is specified explicitly. To formalize the MADM problem, let the set of alternatives X be defined in terms of decision variables; that is, $\mathbf{X} = \{\mathbf{x}_{i*} \mid i = 1,2,\ldots,m\}$. The alternatives are represented by the set of cells or pixels in a raster GIS database or a set of points, lines, or/and areal objects in a vector GIS. Thus the index i indicates the location of the ith alternative. For the sake of simplicity we use a single subscript to indicate the location of an alternative. Thus each alternative is described by means of its locational attribute (coordinate data) and attribute data (criterion values). Since the attributes serve as decision variables, we can designate a criterion outcome (criterion value) by x_{ij}, which represents the level of the jth attribute with respect to alternative i. Hence an alternative i can be characterized by the vector

$$\mathbf{x}_{i*} = (x_{i1}, x_{i2}, \ldots, x_{in}) \qquad \text{for } i = 1,2,\ldots,m \tag{3.1}$$

and the levels of attributes across an alternative are represented by the vector

$$\mathbf{x}_{j*} = (x_{1j}, x_{2j}, \ldots, x_{mj}) \qquad \text{for } j = 1,2,\ldots,n \tag{3.2}$$

The input data for spatial MADM [equations (3.1) and (3.2)] can be organized in tabular form (Table 3.2). The table, also referred to as a *decision, evaluation*, or *impact matrix*, shows the alternative–attribute relationships. The rows of the matrix represent the alternatives (geographical entities). Each alternative is described by its locational (coordinate) data and attribute data or attributes. Each attribute accounts for a column in the decision matrix for the MADM problem. It is usually desirable that the column labels represent "independent" qualities in the colloquial sense of the term; that is, the entries in one column are not predictably related to those in another by the inherent structure or formulation of the problem. The cells of the matrix contain the measured or assessed values of attributes with respect to the alternatives. Notice

TABLE 3.2 Matrix of the Attribute–Alternative Relation for a MADM Problem[a]

	Attribute 1	***Attribute 2***	...	***Attribute n***
Alternative 1	x_{11}	x_{12}	...	x_{1n}
Alternative 2	x_{21}	x_{22}	...	x_{2n}
...	...	...	...	...
Alternative m	x_{m1}	x_{m2}	...	x_{mn}

[a] x_{ij}, score for the ith alternative with respect to the jth attribute ($i = 1,2,\ldots, m$; $j = 1,2,\ldots,n$).

that the matrix has a structure similar to that of the geographical data matrix (see Section 1.1). The input data to a spatial multiattribute problem have a commonly used map layer structure (see Figure 2.5). The data consist of a set of n data layers, and each object in the data layer contains an attribute value. Each object (e.g., raster or polygon) in the map layer can be considered as a decision alternative, or the alternatives can be determined as a combination of objects (points, lines, and/or polygons). In a particular decision situation the set of alternatives can be limited by imposing constraints on the attribute values (aspatial constraints) or on the locational attributes (spatial constraints). For example, all rasters containing attribute values (e.g., slope) greater than some threshold (e.g., 20%) can be eliminated from the set of feasible alternatives, or cells/alternatives located within 25 km of a highway can be excluded from the set of feasible alternatives.

Given the input data, the problem is to aggregate the map layers according to a decision rule so that the "best" alternative can be selected. The performance of an alternative depends not only on the level of the attribute by which an alternative is characterized but also involves the decision maker's preferences with respect to the evaluation criteria (attributes). The preferences are contained in the decision rule. This means that the criterion (decision) outcomes combine the value of the evaluation criteria and the preferences assigned to the criteria. Hence, in most general terms, the MADM problem can be defined as follows: Decision rule:

$$\{x_{i1}, x_{i2}, \ldots, x_{in} \mid \mathbf{x}_{i^*} \in \boldsymbol{X},\ i = 1,2,\ldots,m\} \tag{3.3}$$

This expression can be interpreted as follows: Apply the decision rule to choose the best alternative (to order the alternatives $\mathbf{x}_{i^*}$) in the set of feasible alternatives $\mathbf{X}$, according to the values of the n attributes (see Chankong and Haimes 1983).

Unlike MADM approaches, the MODM methods make a distinction between the concept decision variables and the decision criteria (see Section 3.2.1). These two elements are related to one another by a set of objective functions. Also, the set of alternatives is defined in terms of causal relationships and constraints on the decision variables. The alternatives are implicitly defined rather than given explicitly as in the case of MADM. From the MODM perspective the attributes can be viewed as means or information sources available to the decision maker for formulating and achieving his or her objectives (Starr and Zeleny 1977). In other words, the objectives are functionally related to or derived from some of the attributes. Consequently, the input data to spatial MODM problems can be stored in GISs in the form of map layers. Each map layer contains a set of objects that are considered as elements of an alternative. The alternatives are derived from the map layers by defining the relationship between the objectives and the underlying attributes of the objects contained in geographical space. Since the relationships are defined implicitly as decision variables assigned to objects, the alternatives have to be generated. The input map layers have to be processed to obtain a set of alternatives. It is important to note that the process typically requires an algorithm specifically designed to tackle MODM problems. Usually, it is not possible to use the standard (fundamental) set of operations available in GISs to generate the spatial multiobjective alternatives (see Chapter 2 for a discussion of fundamental operations). The spatial multiobjective analysis goes far beyond the standard GIS tools, such as map overlay techniques (Macmillan and Pierce 1994; Bender and Simonovic 1995; Malczewski and Ogryczak 1996).

To formalize the MODM problem, let us denote an objective by k ($k = 1,2,...,q$). It is assumed that the decision maker's objectives are functionally related to their underlying attributes, that is, $k \in \{1,2,...,n\}$. If the attribute–objective relationship is represented by $f_{ij} = f_j(x_{i*})$, we can define the following vectors:

$$\mathbf{f}_{i*} = (f_{i1}, f_{i2},...,f_{iq}) \qquad \text{for } i = 1,2,...,m \tag{3.4}$$

$$\mathbf{f}_{*j} = (f_{1j}, f_{2j},...,f_{mj}) \qquad \text{for } j = 1,2,...,q \tag{3.5}$$

Equation (3.4) indicates that each alternative, i, is evaluated on the basis of a set of objectives that are functionally related to the underlying attributes. On the other hand, the value of the objective function, f_{*j}, across all alternatives is represented by the vector in equation (3.5) (Starr and Zeleny 1977). Similar to MADM, the MODM problems can be represented in the form of a matrix (Table 3.3). The matrix represents typical objective–alternative relationships for the MODM problem. The row of the matrix shows alternatives, and the column of the matrix contains objectives. The matrix cells contain the objective functions that describe the alternatives in terms of a set of measured or assessed values of attributes with respect to the alternatives. Notice that in the MODM analysis the attributes can be organized in GISs using the map layer structure (see Figure 2.5).

Like MADM, the MODM problem involves the decision maker's preferences. The preferences are contained in the multiobjective decision rule that combines the input data (geographical data and data on decision maker's preferences) into a composite score (criterion or objective outcomes) with respect to each feasible alternative. Given a decision rule, the MODM problem

TABLE 3.3 Matrix of the Objective–Alternative Relation for a MODM Problem

	Objective 1	***Objective 2***	...	***Objective q***
Alternative 1	f_{11}	f_{12}	...	f_{1q}
Alternative 2	f_{21}	f_{22}	...	f_{2q}
...	...	...	...	...
Alternative m	f_{m1}	f_{m2}	...	f_{mq}

involves finding the "best" alternative (or ranking the alternatives) in the set of feasible alternatives **X**, according to the values of the objective functions. Formally:
Decision rule:

$$|f_{i1}, f_{i2},...,f_{iq} \mid \mathbf{x}_{i^*} \in X, \; i = 1,2,...,m| \tag{3.6}$$

This expression can be interpreted as follows: Apply the decision rule to choose the best alternative (order the alternatives $\mathbf{x}_{i^*}$) in the set of feasible alternatives **X**, according to the values of the objective functions.

3.3.2 Framework

Decision making is a process. It involves a sequence of activities that starts with decision problem recognition and ends with recommendations. It is argued that the quality of the decision making depends on the sequence in which the activities are undertaken. There are a number of alternative ways to organize the sequence of activities in the decision-making process. According to Keeney (1992), two major approaches include the *alternative-focus approach*, which focuses on generating of decision alternatives, and the *value-focused approach*, which uses the values (evaluation criteria) as the fundamental element of the decision analysis. The sequence of activities involved in these two approaches is given in Table 3.4. Comparing these two approaches, we can see that the differences between them are related to the question of whether alternatives should be generated first and then the value structure should be specified, or conversely, the alternatives are derived from the value structure. The general principle for structuring the decision-making process is that the decision alternatives should be generated so that the values specified for the decision situation are best achieved (Keeney 1992). This implies that the order of thinking focuses first on what is desired and then on alternatives to obtain it. It is argued that

TABLE 3.4 Comparing Sequences of Activities for the Value- and Alternative-Focused Approaches

Step	*Value-Focused Approach*	*Alternative-Focused Approach*
1	Decision problem recognition	Decision problem recognition
2	Specify values	Identify alternatives
3	Generate alternatives	Specify values
4	Evaluate alternatives	Evaluate alternatives
5	Select an alternative	Select an alternative
6	Recommendation	Recommendation

Source: Based on Keeney (1992, p. 49).

values are more fundamental than alternatives to a decision problem. In other words, alternatives are the means to achieve the more fundamental values.

What follows is a brief discussion of a framework that is organized in terms of the sequence of activities involved in spatial multicriteria decision analysis. The framework is shown in Figure 3.4. It integrates the phase model of decision making (see Section 2.3) and the major elements of MCDM (see Section 3.1). The framework is based on the value-focused approach. To this end, it is worthwhile to notice that the value structure (the decision maker's goal, objectives, attributes, and associated preferences) is represented by the hierarchical organization of the columns, while the alternatives are represented in the rows of the decision matrix for MCDM (see Figure 3.1).

PROBLEM DEFINITION

Any decision-making process begins with the recognition and definition of the decision problem. Broadly defined, the decision problem is a perceived difference between the desired and existing states of a system. It is a "gap" between the desired and existing states as viewed by a decision maker. The problem

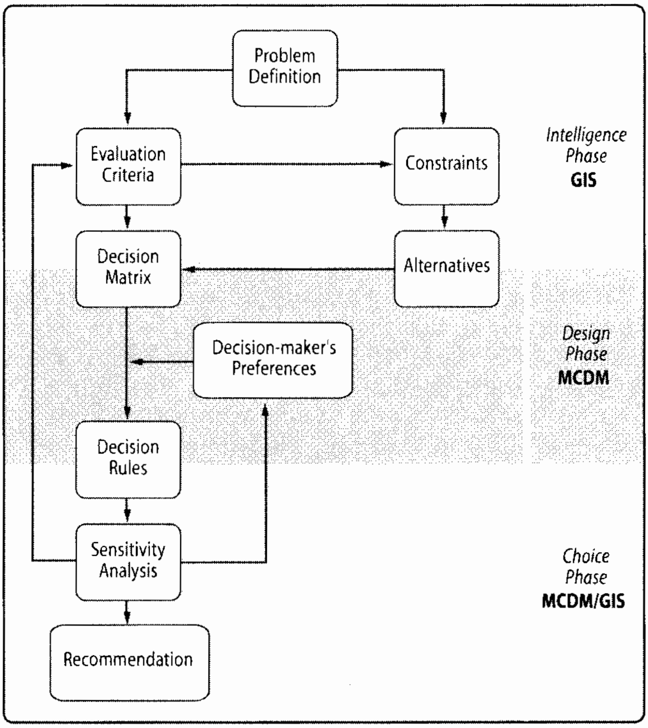

FIGURE 3.4 Framework for spatial multicriteria decision analysis.

definition overlaps the intelligence phase of decision making. In brief, the intelligence phase involves searching the decision environment for conditions calling for decisions; raw data are obtained, processed, and examined for clues that may identify opportunities or problems (see Section 2.3 for details). The GIS capabilities for data storage, management, manipulation, and analysis offer major support in the problem definition stage (see Section 2.2).

EVALUATION CRITERIA

Once the decision problem is identified, the spatial multicriteria analysis focuses on the set of evaluation criteria (objectives and attributes). To be more specific, this step involves specifying (1) a comprehensive set of objectives that reflects all concerns relevant to the decision problem, and (2) measures for achieving those objectives. Such measures are called *attributes.* A measurement scale must be established for each attribute. The degree to which the objectives are met, as measured by the attributes, is the basis for comparing alternatives. The evaluation criteria are associated with geographical entities and relationships between entities and therefore can be represented in the form of maps. There are two types of criterion maps. An *evaluation criterion map* is a unique geographical attribute of the alternative decisions that can be used to evaluate the performance of the alternatives. A *constraint map* displays the limitations on the value that attributes and decision variables may assume. Evaluation criterion maps are also referred to as *attribute maps* (or *thematic maps* or *data layers* in GIS terminology). GIS data-handling and analyzing capabilities are used to generate inputs to spatial multicriteria decision analysis.

ALTERNATIVES

As suggested earlier, the process of generating alternatives should be based on the value structure and be related to the set of evaluation criteria. To each alternative there is assigned a *decision variable.* Variables are used by the decision maker to measure the performance of alternative decisions and in this book will also be called attributes. A set of decision variables defines the *decision space.* Depending on the problem situation, the decision variables may be deterministic, probabilistic, or linguistic. In a real-world situation, very few spatial decision problems can be considered unconstrained. Constraints represent restrictions imposed on the decision space. They determine the set of *feasible alternatives.* In terms of GIS, the constraints are used to eliminate points, lines, polygons and/or rasters characterized by certain attributes and/or certain values of attributes from consideration.

CRITERION WEIGHTS

At this stage, the *decision maker's preferences* with respect to the evaluation criteria are incorporated into the decision model. The preferences are typically expressed in terms of the weights of relative importance assigned to the evaluation criteria under consideration. Broadly speaking, the purpose of criterion (objective or attribute) weights is to express the importance of each criterion relative to other criteria. The derivation of weights is a central step in eliciting the decision maker's preferences. Given the set of alternatives, attributes, and associated weights, the input data can be organized in the form of a decision matrix or table (see Figure 3.1 and Tables 3.2 and 3.3).

DECISION RULES

This step brings together the results of the preceding three steps. Eventually, the unidimensional measurements (geographic data layers) and judgments (preferences and uncertainty) must be integrated to provide an overall assessment of the alternatives. This is accomplished by an appropriate *decision rule* or aggregation function. It is the decision rules that dictate how best to rank alternatives or to decide which alternative is preferred to another. Specifically, the decision rule orders the decision space by means of a one-to-one or one-to-many relationship of outcomes to decision alternatives. This means that a given course of action (alternative) has a given and certain consequence (one-to-one relationship) or uncertain consequences (one-to-many relationship). A consequence is a result of a decision taken by the decision maker. It is sometimes referred to as a *decision outcome* or *criterion outcome*. Accordingly, the set of decision consequences forms the *decision (criterion) outcome space*. Since a decision rule provides an ordering of all alternatives according to their performance with respect to the set of evaluation criteria, the decision problem depends on the selection of the best outcome (or an ordered set of outcomes) and the identification of the decision alternative (or alternatives) yielding this outcome (or outcomes).

SENSITIVITY ANALYSIS

Subsequent to obtaining a ranking of alternatives, *sensitivity analysis* should be performed to determine robustness. Sensitivity analysis is defined as a procedure for determining how the recommended course of action is affected by changes in the inputs of the analysis. To be more specific, it aims at identifying the effects of changes in the inputs (geographical data and the decision maker's preference) on the outputs (ranking of alternatives). If the changes do not significantly affect the outputs, the ranking is considered to be robust. If the current result is found to be unsatisfactory, we may use information about the output to return to the problem formulation step. The sensitivity analysis can be thought of as an exploratory process by which the decision makers achieve a deeper understanding of the structure of the problem. It helps to learn how the various decision elements interact to determine the most preferred alternative and which elements are important sources of disagreement among decision makers or interest groups.

RECOMMENDATION

The end result of a decision-making process is a *recommendation* for future action. The recommendation should be based on the ranking of alternatives and sensitivity analysis. It may include the description of the best alternative or a group of alternatives considered candidates for implementation. Visualization techniques are of major importance in presenting and communicating the results to decision makers and interest groups. The solutions to spatial multicriteria decision problems should be presented in both decision (geographical) space and criterion outcome space.

Although each stage of the spatial multicriteria analysis involves both GIS and MCDM methodologies, the stages differ in terms of the degree to which these two methodologies are used. In the earlier stages, GIS techniques play the major role (see Figure 3.4), while in the latter stages, MCDM techniques are of major importance. This is related to the support offered by GISs and MCDM during the process of making a spatial decision. The extent to which GISs support the three major phases of decision making (i.e., intelligence, design, and choice) has been discussed in Section 2.3. We have also emphasized that a GIS should be considered as a special-purpose digital database in which a common spatial coordinate system is the primary means of storing and accessing data and processing the data to obtain information for decision making and that an ultimate aim of a GIS is to provide support for making decisions. This can be achieved by integrating the MCDM and GIS capabilities. MCDM provides a methodology for guiding decision maker(s) through the critical process of clarifying evaluation criteria (attributes and/or objectives) and of defining values that are relevant to the decision situation. When spatial decision making typically involves a large number of alternatives evaluated on the basis of multiple and conflicting criteria, some systematic method of identifying the best alternatives (of classifying or ranking the alternatives) is required. MCDM methods are designed to help the decision maker under these conditions. They provide the means of performing complex trade-offs on multiple evaluation criteria while taking the decision maker's preferences into account.

SUMMARY AND CONCLUSIONS

In this chapter we have reviewed and classified the key components of MCDM problems. In the most general terms, MCDM problems involve a set of alternatives that are evaluated on the basis of conflicting and incommensurate criteria. A criterion is a generic term that includes both the concepts of attribute and objective. Accordingly, two broad classes of MCDM can be distinguished: MADM (multiattribute decision making) and MODM (multiobjective decision making). The complexity of a particular MCDM (MADM or MODM) problem depends on the number of people (interest groups, decision makers) involved in the decision-making process and the data and information available to tackle the decision problem. To this end, MCDM can be categorized as single-decision-maker problems and group decision problems, and these two categories can, in turn, be subdivided into deterministic, probabilistic, and fuzzy decisions. The deterministic decision problems assume that the required data and information are known with certainty and that there is a known deterministic relationship between every decision and the corresponding decision consequence. Probabilistic analysis deals with the decision situation under uncertainty about the state of the environment and about the relationships between the decision and its consequences. Whereas probabilistic analysis treats uncertainty as randomness and likelihood, fuzzy set analysis deals with the type of uncertainty associated with imprecise information.

The combination of GIS and MCDM capabilities is of critical importance in spatial multicriteria analysis. GISs provide the capabilities of data acquisition, storage, retrieval, manipulation, and analysis of the data to obtain information for making decisions. However, GIS systems have a limited capability as far as the analysis of the value structure is concerned. The MCDM techniques provide the tools for aggregating the geographical data and the decision maker's preferences into unidimensional value or utility of alternative decisions. Based on a discussion of the conventional MCDM structure, a framework for GIS-based (or spatial) multicriteria decision analysis has been developed. The framework consists of a sequence of elements, including problem definition, evaluation criteria (objectives and/or attributes), alternatives, constraint maps, decision-maker preferences, decision rules, sensitivity analysis, and recommendation. In the following chapters of Part II, the components of spatial multicriteria analysis are considered in depth.

REVIEW QUESTIONS

1. Classify MCDM problems according to the meaning of evaluation criteria, the number of decision makers involved in the decision-making process, and the amount of information available to decision makers.
2. Discuss the major differences between conventional MCDM and spatial MCDM.
3. Define the concepts of MCDM, MODM, and MADM.
4. Define the major elements of spatial multicriteria analysis. Identify the role of GIS and MCDM techniques in each stage of the analysis.
5. Compare the decision matrix, geographical data matrix, and spatial interaction matrix (see Figures 1.1 and 3.1). Identify the similarities and differences. Why is it useful to organize the input data for multicriteria decision analysis in a matrix format?